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GRADE 4 • MODULE 4

Angle Measure and Plane Figures

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OVERVIEW

This 20-day module introduces points, lines, line segments, rays, and angles, as well as the relationships between them. Students construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. With angle measure playing a key role in the work throughout the module, students learn how to create and measure angles, as well as how to create and solve equations to find unknown angle measures. In these problems, where the unknown angle is represented by a letter, students explore both measuring the unknown angle with a protractor and reasoning through the solving of an equation. This connection between the measurement tool and the numerical work lays an important foundation for success with future geometry and algebra. Through decomposition and composition activities, as well as an exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures.

Topic A begins with students drawing points, lines, line segments, and rays, as well as identifying these in various contexts and within familiar figures. Students recognize that two rays sharing a common endpoint form an angle (**4.6A**). They create right angles through a paper-folding activity, identify right angles in their environment, and see that one angle can be greater (obtuse) or less (acute) than a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines as they define, draw, and recognize intersecting, perpendicular, and parallel lines (**4.6A**).

In Topic B, students explore the definition of degree measure, beginning with a circular protractor. By dividing the circumference of a circle into 360 equal parts, they recognize one part as representing 1 degree (**4.7A, 4.7B**). Through exploration, students realize that, although the size of a circle may change, an angle spans an arc, representing a constant fraction of the circumference. By carefully distinguishing the attribute of degree measure from that of length measure, the common misconception that degrees are a measure of length is avoided. Armed with their understanding of the degree as a unit of measure, students use various types of protractors to measure angles to the nearest degree and to sketch angles of a given measure (**4.7C, 4.7D**). The idea that an angle measures the amount of *turning* in a particular direction and the notion that the measure of an angle represents a fraction of a circle that is *cut out* are explored as students recognize familiar angles in varied contexts (**4.6A, 4.7A, 4.7B**).

Handwritten annotations:
- construct classify recognize solve define
- ✱ huge vocab!
- Protractor + equation
- — Create + meas. ∠
- — solve unkn
- compare to rt ∠
- (A) Draw: — points — lines — segments — rays
- ② Recog / ident
- inter./para/perp
- Don't just tell!
- ① intro degree w/ cir. protract
- ② use various protractors
- New Arc!
- STAMP
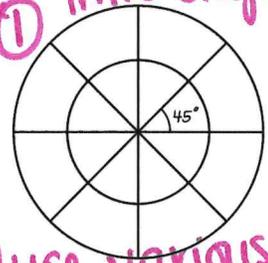
Topic C begins by decomposing 360° using pattern blocks, allowing students to see that a group of angles meeting at a point with no spaces or overlaps add up to 360°. With this new understanding, students now discover that the combined measure of two adjacent angles on a line is 180°, that the combined measure of two adjacent angles meeting to form a right angle is 90°, and that vertically opposite angles have the same measure. These properties are then used to solve unknown angle problems (**4.7E**). The terms "supplementary angle" and "complementary angle" are included in order to expose students to this vocabulary but are not assessed.

An introduction to symmetry opens Topic D as students recognize lines of symmetry for two-dimensional figures, identify line-symmetric figures, and draw lines of symmetry (**4.6B**). Given one half of a line-symmetric figure and the line of symmetry, students draw the other half of the figure. This leads to their work with triangles. Students are introduced to the precise definition of a triangle and then classify triangles based on angle measure (**4.6C**). Students construct triangles given a set of classifying criteria (e.g., create a triangle that is both right and isosceles). Finally, students explore quadrilaterals and classify them based on their attributes, including angle measure and parallel and perpendicular sides (**4.6D**). This work builds on Grade 3 reasoning about the attributes of shapes and lays a foundation for hierarchical classification of two-dimensional figures in Grade 5. The topic concludes as students explore two-dimensional figures and reason about their attributes.

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Throughout Module 4 students engage with the TEKS mathematical process standards by creating representations of various geometrical figures, such as points, lines, line segments, rays, and angles (**4.1E**). Next, students progress to understanding degree measures, and they select and use various protractors to draw and measure angles (**4.1C**). As students discover the additive nature of angle measurement by using concrete examples, they measure unknown angles with a protractor and determine their measures by reasoning through writing addition and subtraction equations. They make connections and explain the relationships between these methods for measuring angles (**4.1F**). In the final topic of the module, students classify two-dimensional figures based on given attributes, and they explain and justify their classifications using precise mathematical language learned throughout the module (**4.1G**).

Collaboratively Troubleshooting Student Misconceptions

It is common for students to make mistakes as they build their understanding of new or difficult concepts. As noted in the Program and Implementation Guide, *collaborative troubleshooting* is a routine to help teachers address students' misconceptions. The three steps to collaborative troubleshooting are
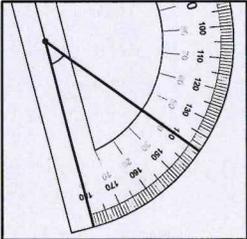
(1) surface student thinking;
(2) validate what the student did right; and
(3) bridge to a better understanding.

The following table presents teachers with guidance on how to collaboratively troubleshoot misconceptions with students. The first three columns of the table outline misconceptions that commonly arise in this module, reasons why students may have the misconceptions, and associated TEKS. Teachers can use this information to help them decide which questions to ask students to surface thinking and to validate what the students understood or did correctly.
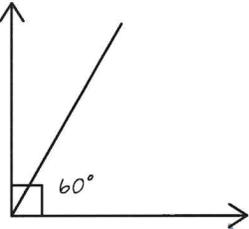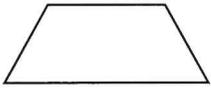
The last column of the table provides instructional strategies and sample guided questions that can support students as they build on what they already know and bridge to a better understanding.

Note: Teachers can also refer to the sample teacher–student dialogue in the "Collaboratively Troubleshooting Student Misconceptions" section of the Grade 4 Course Guide for additional guidance on implementing the three-step routine.

Topic	TEKS	Student Misconception	How to Bridge to a Better Understanding
Topic A	4.6A	Students use the familiar, everyday definition of new terminology instead of the precise mathematical definition (e.g., "A line has a start and end because when I line up for recess, there is a front and back of the line."), or use the definition of another term introduced at the same time (e.g., "Parallel lines intersect and form a right angle, but perpendicular lines never intersect").	Continue to engage in activities that solidify learning of the new terms. For example, ■ Display labeled images of each term for students to refer to. ■ Find representations of new terms in nearby objects. ■ Use the Physiometry fluency activities to relate terminology to kinesthetic movements. ■ Provide discussion prompts that encourage partners to use the terminology in conversation.
Topic B	4.7C 4.7D	Students using a protractor start measuring at a place other than 0° or read from the other set of numbers when there are two (e.g., "I see 40° and 140° on the protractor. This angle measures 140°").	Relate measuring with a protractor to students' background knowledge of measurement and estimation. For example, ■ Relate to starting at 0 when measuring length with a ruler. ■ Point to 0 and then trace along the edge of the protractor while counting up. ■ Identify the type of angle first, and then assess the reasonableness of the measurement based on the angle type.

(handwritten annotation on Topic A): push academic vocab

(handwritten annotation on Topic B): push correct steps to measure

Topic	TEKS	Student Misconception	How to Bridge to a Better Understanding
Topic C	4.7E	Students are unsure whether to add or subtract angle measures when calculating to find an unknown angle measure (e.g., 60 + 90 = _____).	Notice how part–part–whole language is used to describe the angle measures. Connect to student experience with part–part–whole relationships in whole number addition and subtraction. When writing an equation to solve, identify the whole angle measurement, the parts an angle has been broken into, and the information that is known and unknown.
Topic D	4.6C 4.6D	Students see attributes of a polygon together without isolating individual angles or pairs of sides to classify the polygon (e.g., "None of the sides of this quadrilateral are parallel because they are all intersecting with another side").	Use available tools to focus on specific sides and angles within a polygon. For example, ▪ Overlap the right-angle tool on an angle in a polygon to estimate the angle measure. ▪ Use a straightedge to trace and extend line segments in a pair of sides or an angle so the line segments stand out from the rest of the polygon. ▪ Use colored pencils or highlighters to trace and identify attributes in a polygon. ▪ Insert pages into a personal white board to trace and extend attributes of the polygon with a marker.

Handwritten annotations: "Connect to part-part-whole" · "60°" · "poss strip diag?" · "Ahh! I see this!"

Focus Grade Level Standards

Geometry and Measurement

The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

SS. **4.6A**   identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;

SS. **4.6B**   identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;

SS. **4.6C**   apply knowledge of right angles to identify acute, right, and obtuse triangles;

RS. **4.6D**   classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Geometry and Measurement

The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

can't be tested

4.7A   illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;

4.7B   illustrate degrees as the units used to measure an angle, where $\frac{1}{360}$ of any circle is one degree and an angle that "cuts" $\frac{n}{360}$ out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;

RS. **4.7C**   determine the approximate measures of angles in degrees to the nearest whole number using a protractor;

SS. **4.7D**   draw an angle with a given measure;

SS. **4.7E**   determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.

Foundational Standards

The student is expected to:

3.4K   solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts;

3.5A	represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;
3.5B	represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations.

TEKS Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

4.1C	select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
4.1E	create and use representations to organize, record, and communicate mathematical ideas;
4.1F	analyze mathematical relationships to connect and communicate mathematical ideas;
4.1G	display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Overview of Module Topics and Lesson Objectives

TEKS	ELPS		Topics and Objectives	Days
4.6A 4.6C	1.F 2.C 2.E 2.I 3.D 3.F 4.F 5.G	A	**Lines and Angles** Lesson 1:　Identify and draw points, lines, line segments, rays, and angles. Recognize them in various contexts and familiar figures. Lesson 2:　Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles. Lesson 3:　Identify, define, and draw perpendicular lines. Lesson 4:　Identify, define, and draw parallel lines.	4

(handwritten notes: "Intro to vocab", "start ∠'s", "→ Lines")

TEKS	ELPS		Topics and Objectives	Days
4.7A **4.7B** **4.7C** **4.7D** 4.6A	1.E 2.C 2.E 2.I 3.E 4.F 5.G	B	**Angle Measurement** Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of any circle. Explore benchmark angles using the protractor. Lesson 6: Use varied protractors to distinguish angle measure from length measurement. Lesson 7: Measure and draw angles. Sketch given angle measures, and verify with a protractor. Lesson 8: Identify and measure angles as turns and recognize them in various contexts.	4
			Mid-Module Assessment Task: Topics A–B	2
4.7E	2.C 2.D 2.E 2.I 3.E 3.H 4.D 4.F 5.G	C	**Problem Solving with the Addition of Angle Measures** Lesson 9: Decompose angles using pattern blocks. Lessons 10–11: Use the addition of adjacent angle measures to solve problems using a letter for the unknown angle measure.	3
4.6A **4.6B** **4.6C** **4.6D**	1.E 2.C 2.E 2.F 2.I 3.E 4.F 5.G	D	**Two-Dimensional Figures and Symmetry** Lesson 12: Recognize lines of symmetry for given two-dimensional figures. Identify line-symmetric figures, and draw lines of symmetry. Lesson 13: Analyze and classify triangles based on angle measure. Lesson 14: Define and construct triangles from given criteria. Lesson 15: Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size. Lesson 16: Classify two-dimensional figures based on parallel and perpendicular lines and the presence or absence of angles of a specified size.	5
			End-of-Module Assessment Task: Topics A–D	2
Total Number of Instructional Days				20

Handwritten annotations: circle prot. · benchmark angles · more prot. · meas + draw · ident + meas. · super hands on! · solve unknown · symmetry · classify △ · Define/construct△ · Quads · EXACT TEK! · All 2D

Terminology

Tons of cognates

A Spanish cognate is included when the term has a similar meaning and spelling in English. Not every term in this module has a Spanish cognate.

New or Recently Introduced Terms

- **Acute angle** (*Ángulo agudo*): an angle that measures less than 90 degrees
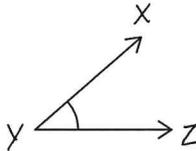
- **Acute triangle** (*Triángulo agudo*): a triangle in which each of the three angles is acute (less than 90°)
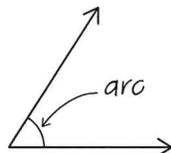

acute triangle
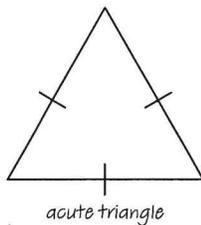
- **Adjacent angle** (*Ángulo adyacente*): two angles with the same vertex that share a ray and do not overlap (e.g., $\angle URT$ and $\angle TRS$ are adjacent angles)
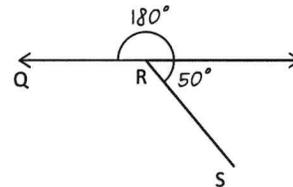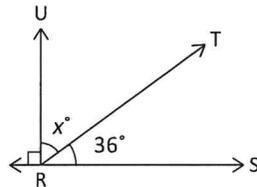
- **Angle** (*Ángulo*): the union of two rays sharing a common vertex[1]
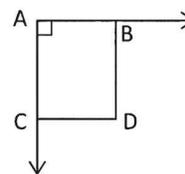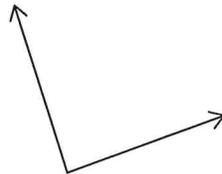
- **Arc** (*Arco*): a portion of the circumference of a circle; an arc can be used to mark an angle

arc

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[1] See margin note on angles in lesson 5 for additional explanation of angles that are represented with line segments.

- **Collinear** (*Colineal*): points are collinear if there is a line containing all of the points; if there is no such line, the points are non-collinear
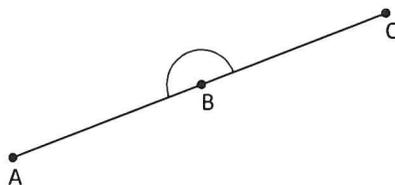
- **Complementary angles** (*Ángulos complementarios*): two angles whose measures sum to 90 degrees

complementary angles
$$30° + 60° = 90°$$
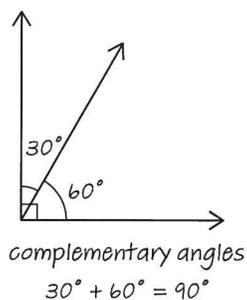
- **Degree**: the unit used to measure an angle, where an angle that is $\frac{1}{360}$ of a whole turn of a circle has a measure of 1 degree (°)
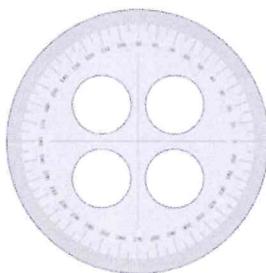
- **Degree measure of an angle:** the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle; an angle that is $\frac{n}{360}$ of a whole turn of a circle has a measure of n degrees
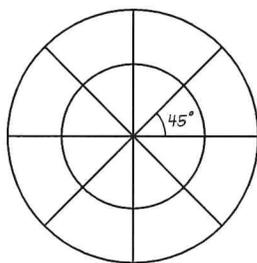
- **Diagonal** (*Diagonal*)**:** a line segment, ray, or line that is not vertical or horizontal

- **Figure** (*Figura*): a set of points in the plane
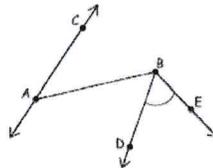
- **Interior of an angle** (*Interior de un ángulo*): the convex[2] region defined by the angle
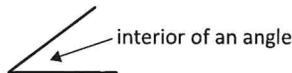
- **Intersecting lines:** lines that contain at least one point in common
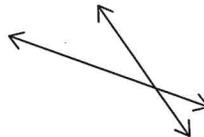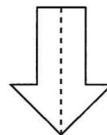

- **Length of an arc:** distance along the arc
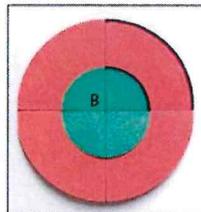
- **Line** (*Línea*): a straight path with no thickness that extends in both directions without end
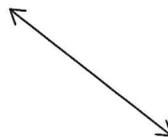
- **Line of symmetry** (*Línea de simetría*): a line dividing an image into two congruent parts that are mirror images of each other

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[2] In Grade 4, a picture will suffice. A precise definition of convexity is given in high school geometry.

- **Line segment** (*Segmento de línea*): a part of a line with two end points and the set of points on the line between them

- **Obtuse angle** (*Ángulo obtuso*): an angle that measures more than a right angle (90 degrees) and measures less than a straight angle (180 degrees)
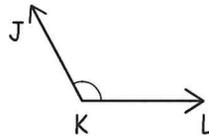
- **Obtuse triangle** (*Triángulo obtuso*): a triangle with an interior angle measuring more than 90 degrees
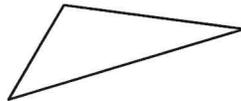
- **Parallel** (*Paralelo*): two lines in a plane that do not intersect and are always the same distance apart; line segments or rays are parallel if the lines containing them are parallel
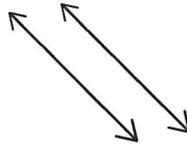
- **Perpendicular** (*Perpendicular*): two lines that intersect if the angles formed between the lines are right angles; line segments or rays are perpendicular if the lines containing them are perpendicular
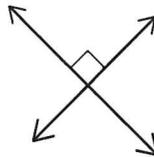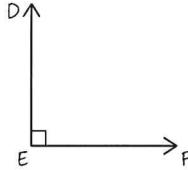
- **Point** (*Punto*): a precise location in the plane

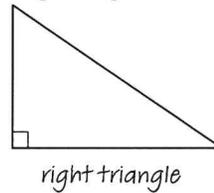- **Protractor**: an instrument used in measuring or sketching angles
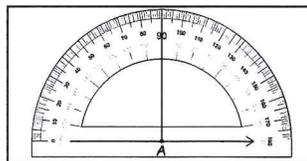
- **Ray** (*Rayo*): a point and the set of all points on the line in one direction away from the point

▪ **Right angle:** an angle formed by perpendicular lines, measuring 90°

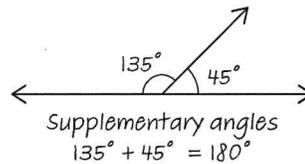▪ **Right triangle:** a triangle that contains one right angle

right triangle

▪ **Straight angle:** an angle that measures 180°

▪ **Supplementary angles** (*Ángulos suplementarios*): two angles whose measures sum to 180 degrees

Supplementary angles
135° + 45° = 180°

▪ **Triangle** (*Triángulo*): three non-collinear points and the three line segments between them; the three line segments are called the *sides* of the triangle, and the three points are called the *vertices*
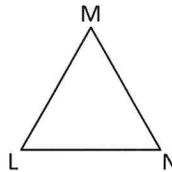
▪ **Vertex** (*Vértice*): a point where two lines, line segments, rays, or sides meet
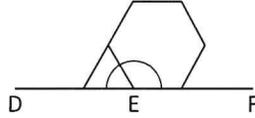
vertex
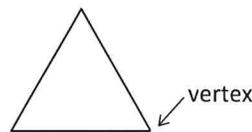
▪ **Vertical angles** (*Ángulos verticales*): When two lines intersect, any two non-adjacent angles formed by those lines are called *vertical angles* or *vertically opposite angles*.
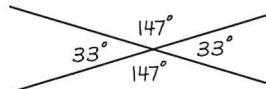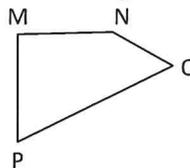
Familiar Terms and Symbols[3]

- **Decompose** (*Descomponer*): the process of separating into smaller components (e.g., to break a larger angle into smaller angles)

    D   E   F

- **Parallelogram** (*Paralelogramo*): a quadrilateral with two pairs of parallel sides

    Parallelogram

- **Polygon** (*Polígono*): a closed two-dimensional figure with straight sides

    M   N
            O
    P

- **Quadrilateral** (*Cuadrilátero*): a polygon with four sides

    A
    Z   W
        F
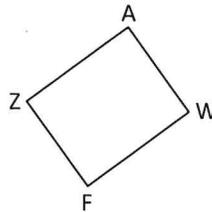
- **Rectangle** (*Rectángulo*) a quadrilateral with four right angles

    A   B
    C   D
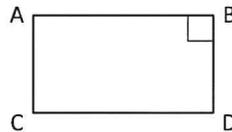
- **Rhombus** (*Rombo*): a quadrilateral with all sides of equal length

    Rhombus

- **Square:** a rectangle with all sides of equal length

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[3] These are terms and symbols students have used or seen previously.

- **Sum** (*Suma*): the result of adding two or more numbers

sum

$$60° + 60° + 60° + 60° + 60° + 60° = 360°$$

- **Trapezoid** (*Trapezoide/trapecio*): a quadrilateral with exactly one pair of parallel sides

Suggested Tools and Representations

- Folded paper models
- Pattern blocks
- Protractors of various diameters, including a 360° and 180° protractor
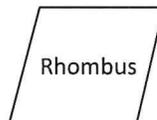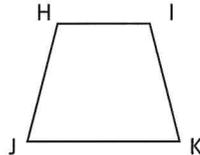- Rectangular and triangular grid paper
- Right angle template (created in Lesson 2), set square
- Ruler (used to measure length), straightedge (used to draw straight lines)

Module 4 Lesson Overview Materials List

Lesson	Teacher Materials	Student Materials All counts are per student unless otherwise indicated.
1	Straightedge	Personal white board Straightedge Blank paper
2	Blank paper Straightedge Right angle (T)*	Personal white board Blank paper Straightedge Right angle (T)
3	Straightedge Right angle (T) Paper	Personal white board Straightedge Right angle (T) Paper
4	Personal white board Straightedge Square grid paper Right angle (T)	Personal white board Straightedge Square grid paper Right angle (T)

Lesson	Teacher Materials	Student Materials *All counts are per student unless otherwise indicated.*
5	2 Paper circles Circular protractor (T)	Personal white board Straightedge 2 Paper circles Right angle (T) Circular protractor (T)
6	Circles (T) 2 pieces of wire or yarn Practice Sheet (T) Dark marker Straightedge Assortment of protractors (at least one circular protractor and one 180° protractor)	Personal white board Straightedge Circles (T) Practice Sheet (T) Dark marker Assortment of protractors (at least one circular protractor and one 180° protractor)
7	Circular protractor 180° protractor Practice Sheet (T)	Personal white board Circular protractor 180° protractor Practice Sheet (T)
8	Analog clock	Personal white board Clock (T)
9	Pattern blocks Straightedge Protractor	Personal white board Pattern blocks Straightedge Protractor
10	Blank paper Personal white board Straightedge Protractor Pattern blocks	Blank paper Personal white board Straightedge Protractor Pattern blocks
11	Blank paper Personal white board Straightedge Protractor Pattern blocks Red marker Blue marker Chart of pattern block angle measures	Blank paper Personal white board Straightedge Protractor Pattern blocks Red and blue pencils, markers, or crayons

Lesson	Teacher Materials	Student Materials *All counts are per student unless otherwise indicated.*
12	Pentagon (T) 1 paper cutout of each of the following shapes: rectangle, square, parallelogram, rhombus, trapezoid, and circle, Lines of symmetry (T)	Personal white board Pentagon (T) Paper cutout of 1 rectangle and 1 square (per pair) Straightedge Lines of symmetry (T)
13	Triangles (T) Practice Sheet (T) Graph paper Ruler	Personal white board Triangles (T) one set per group Practice Sheet (T) Graph paper Ruler Protractor
14	Ruler Protractor Blank paper	Personal white board Square grid paper Ruler Protractor Blank paper
15	Right angle template (T) Quadrilaterals (T) Quadrilateral table (T)	Personal white board Square grid paper Right angle template (T) Quadrilaterals (T) Quadrilateral table (T)
16	Personal white board Rules page (T) Guess My Rule game board (T) Polygons for game (T)	Personal white board Rules page (T) one per group Guess My Rule game board (T) one per group Polygons for game (T) one per group

*(T) Template provided in TE, Practice, and/or Learn